Noise and synchronization in chaotic neural networks

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We show that two identical fully connected chaotic neural networks can always achieve a stochastic synchronization state when linked with a sufficiently large common noise. This is the case for both low-dimensional hyperchaos and high-dimensional spatiotemporal chaos. When the parameters of the two driven systems possess a tiny difference, weakly noise-induced synchronization is obtained. Unstable finite-precision synchronization of chaos with positive conditional Lyapunov exponent is also observed. It is caused by the on-off synchronizing intermittent dynamics. [S1063-651X(98)10106-X]

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It is well known that a criterion for the occurrence of chaotic behavior in deterministic systems is their sensitive dependence on initial conditions. However, theoretical studies and practical experiments show that coupled chaotic systems possess the property of synchronization [1–5]. On the other hand, the effects of additive noise on chaos are studied in various systems. It turns out that, close to bifurcations or crisis and also for chaotic windows, noise tends to amplify chaoticity [6–8]. At parameter values with chaotic dynamics, the Lyapunov exponents are robust against small fluctuations in most cases [9]. The effect of noise on the synchronization of chaotic systems is also studied [10]. It is shown that synchronization will not occur if driving dynamical chaos is added with noise. This matches the common thought that in low-dimensional systems, chaos is favored by small external noise [9].

The system driven by noise is a stochastic one. Can a synchronization state be achieved when two identical systems with slightly different initial conditions are driven by the same noise? This question is closely related to the problem of generalized synchronization [4,11–14]. However, the driving signal used here is noise rather than chaos. Recently, considerable attention has been drawn to the problem of noise-induced synchronization in chaotic systems [15–25]. With these studies and arguments, the effects of noise on chaos are investigated in detail. The conditional Lyapunov exponent (CLE) can be defined for systems driven by noise [15]. With small noise, the driven systems often still possess positive CLE's. Although they cannot achieve a chaotic synchronization state [15,24], two chaotic systems subjected to the same noise with a sufficiently large amplitude have a much higher probability of approaching each other than in the absence of noise [19]. As a result, finite-precision synchronization can be obtained [18,23]. A chaotic attractor means that the overall rate of expansion of the trajectories is larger than that of contraction. Disturbed by a sufficiently large noise, the trajectories are often driven out of this chaotic attractor. In this case, if the system is still stable, it often implies that the overall rate of contraction of the trajectories becomes large. Thus, the CLE's of the driven system may become negative and a stable synchronization state can be obtained. A sufficiently large noise often changes the chaotic nature of the systems and so the resultant attractor is stochastic, rather than chaotic [22]. In other words, the stable synchronizing state is stochastic for systems driven by noise while the generalized synchronization state [4,11–15] is chaotic due to the chaotic driving signal.

There was a common thought that synchronization of hyperchaos cannot be achieved with a scalar driving chaos. However, some approaches for the synchronization of hyperchaos with scalar chaos were proposed recently [26–28]. There are also studies on the synchronization of high-dimensional spatiotemporal chaos with scalar driving chaos [14,29,30]. As the discussions on noise-induced synchronization [15–25] by now are mainly based on simple chaotic systems such as the logistic map or Lorenz equations, an interesting question arises: Can a common scalar noise drive two identical hyperchaotic systems to synchronization? The main objective of this paper is to give a definite answer to this question by using a chaotic neural network [31,32] as an example. Similar to the synchronization of hyperchaos [14,26], one of the possible applications of noise-induced synchronization with a hyperchaotic system is in the field of secure communications.

In the first part of the paper, we will show that two identical low-dimensional hyperchaotic systems driven by a sufficiently large noise can be transformed to a stochastic state and achieve synchronization. Here we will also point out that, when the CLE's of the system driven by noise are positive, it is the on-off synchronizing intermittent dynamics that causes the chaotic trajectories to have a much higher probability to approach each other. As a result, finite-precision synchronization of chaos [18,23] is obtained.

Neural networks are systems formed by a large number of interconnected neurons. For these systems, the corresponding chaotic attractors are often referred to as spatiotemporal ones. In this paper, we will discuss the possibility of the occurrence of noise-induced synchronization in high-dimensional chaotic neural networks. The goal is to understand how scalar noise drives the dynamics of the spatiotemporal chaotic neural networks to a stochastic synchronizing state. This question is related to the recent research interest

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in understanding globally coupled systems driven by noise [33–35]. In Ref. [33], by using the dynamical mean-field equation, it is pointed out that in high-dimensional systems where chaos is due to interactions between nonchaotic neurons, noise can impair the information flow between these neurons and therefore tends to suppress chaos. We will discuss, in the view of synchronization, the case of neural networks consisting of chaotic neurons. Our result entails a much stronger statement that, even for the high-dimensional spatiotemporal complex system, the resultant highly erratic and random trajectories become independent of the initial positions; i.e., they fall into a synchronization state.

Consider a nonmonotonic neural network model that consists of N analog neurons $\{S_i(t)\}$, $i=1,\ldots,N$. Each neuron S_j is connected to all other neurons S_i by couplings J_{ij}. We use parallel dynamics for the updating of neurons:

$$S_i(t+1)=f(h_i(t)),\quad i=1,\ldots,N. \tag{1}$$

Here $h_i(t)$ is the weighted input of the ith neuron and is expressed as

$$h_i(t)=\sum_{j=1}^{N} J_{ij}S_j(t),\quad i=1,\ldots,N. \tag{2}$$

The activation function $f(x)$ of the neurons is nonmonotonic [31,32]:

$$f(x)=\tanh(\alpha x)\exp(-\beta x^2). \tag{3}$$

With the function (3), we have $|S_i|<1$. A significant difference between this and the logistic map investigated in [17,20,21] is that the input space is unbounded for the former map, while it is bounded to $[-1,\,1]$ in the latter case. As a result, a noise with large amplitude often destroys the stable attractor of the logistic map before the stable synchronization state is reached. This is the reason why noise cannot drive chaos to a synchronization state [20,21]. In contrast to this, a function with an infinite input boundary is used in our study and so noise with any amplitude can be applied. Thus, a different result is obtained.

We consider a four-neuron model with parameters $\alpha=6.0$ and $\beta=10.0$ and synaptic connection matrix J:

$$J=\begin{pmatrix} 1.0 & 0.1 & -0.9 & -0.3\\ 0.5 & 0.9 & 0.2 & 0.0\\ -0.7 & 0.4 & -0.3 & 0.1\\ 0.5 & 0.1 & 0.2 & 1.0 \end{pmatrix}. \tag{4}$$

In this case, $|S_i|<0.58$. Simulation results show that the attractor is hyperchaotic with the four Lyapunov exponents being 0.660, 0.353, 0.008, and -0.714 (± 0.002), respectively. If there is no interconnection, i.e., $J=0$, each neuron is in the chaotic state. Now we discuss the simulation results of noise-induced synchronization with this hyperchaos.

Suppose that the corresponding neurons in the two identical neural networks are disturbed by a common random noise η with amplitude η_0. Mathematically,

$$S_i(t)+\eta_0\text{random}(-1,1):\Rightarrow S_i(t). \tag{5}$$

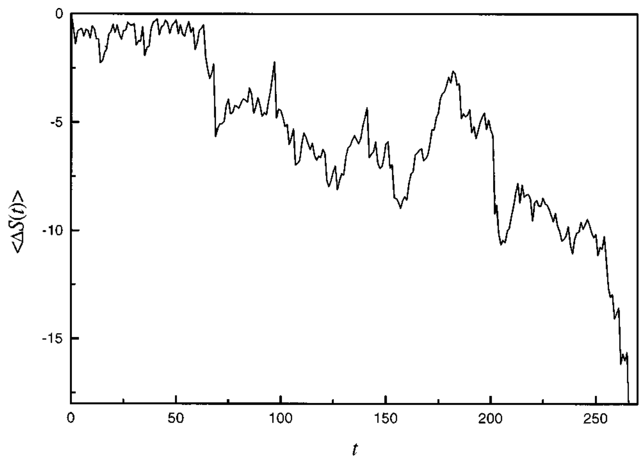

FIG. 1. A plot of time versus the logarithm of the average absolute difference $\langle \Delta S(t)\rangle$ of the two neural networks when the first neuron is driven by noise of amplitude $\eta_0=1.5$.

Computer simulations show that when the first neuron is driven by noise with $\eta_0 \geq 1.09$, synchronization can be achieved. This can also be found in the second and the third neurons with $\eta_0 \geq 4.8$ and 2.3, respectively. To show the distance between the two trajectories, the average absolute difference $\langle \Delta S(t)\rangle$ is defined:

$$\langle \Delta S(t)\rangle=\frac{1}{N}\sum_{i=1}^{N} |S_i'(t)-S_i(t)|. \tag{6}$$

Figure 1 shows a plot of time versus the logarithm of the average absolute difference $\langle \Delta S(t)\rangle$ when the first neuron is driven by noise with $\eta_0=1.3$.

The CLE's can be calculated from the tangent space of the neural dynamics. Again, the first neuron of the chaotic neural network is driven by a common noise term and a plot of the four CLE's versus noise amplitude η_0 is shown in Fig. 2, in which one can see that when $\eta_0>1.244$, the four CLE's are all negative, and so stable hyperchaotic synchronization

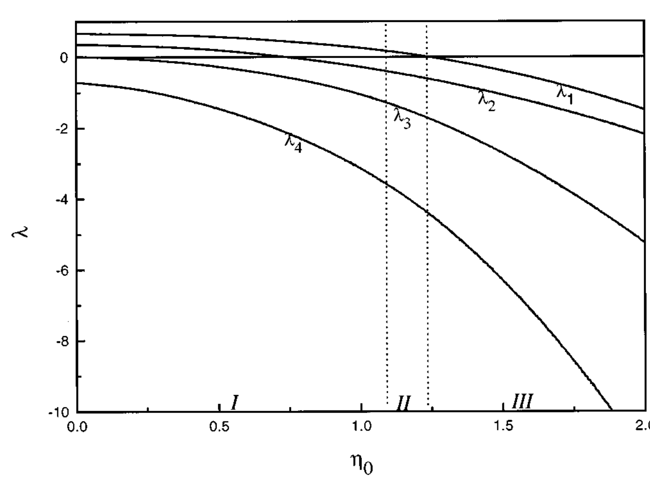

FIG. 2. The four CLE's (λ_i. $i=1,\ldots,4$) of the neural networks versus noise amplitude η_0. Here, region I corresponds to the asychronization state; region II corresponds to the unstable finite-precision synchronization of chaos with positive CLE; region III corresponds to the stable stochastic synchronization state.

can be obtained. For example, the four CLE's are -0.075, -0.713, -1.932, and -4.805, respectively, when $\eta_0=1.3$.

Simulation results also show that synchronization can still be achieved in the interval $1.09<\eta_0<1.244$ where the maximum CLE is positive. For example, when $\eta_0=1.2$, the four CLE's are 0.055, -0.550, -1.595, and -4.173, respectively. This phenomenon is referred to as the finite-precision synchronization of chaos [18,23]. It is pointed out [36] that chaotic systems driven by noise (or chaos) possess the on-off intermittency property if both the following criteria are satisfied: (a) there exists a lower-dimensional invariant hyperplane under the evolution of the system, and (b) there exist orbits entering and leaving every sufficiently small neighborhood of the hyperplane. In fact, the two criteria are naturally obeyed for the difference between the trajectories (i.e., $\Delta S=S'-S$). This is because it is the difference of the driven trajectories, rather than the driven trajectory itself, that constructs an invariant hyperplane, i.e., $\Delta S=0$. Here, we call it the on-off synchronizing intermittency. One can see that the on-off synchronizing intermittency is an intrinsic character of synchronizing systems. As a result of this character, the trajectory can be driven, in a finite time interval, to the invariant hyperplane ($\Delta S=0$), i.e., the contracting region, with high frequency even when the maximum CLE is slightly positive. As a result, the trajectories are driven to approach each other. When the distance between different trajectories is smaller than the finite precision of computer calculations, it is set to zero and remains at that value thereafter. Finite-precision synchronization of chaos is thus achieved. For all the simulations reported in this paper, the synchronization state corresponds to the situation that the difference of two trajectories is smaller than 10^{-18}, which cannot be distinguished in our computer.

In real applications, the synchronization is often affected by the differences in parameters of the two driven systems. To investigate this, we let there be a small deviation $\delta\alpha$ between the corresponding parameters α and α' of the two networks, i.e., $\delta\alpha=\alpha'-\alpha$. The corresponding variational equation is

$$\delta S_i(t+1)=f\left(\alpha',\sum_j J_{ij}S'_j(t)\right)-f\left(\alpha,\sum_j J_{ij}S_j(t)\right)$$
$$=\sum_j \frac{\partial f(\alpha,\sum_j J_{ij}S_j(t))}{\partial S_j}\delta S_i(t)$$
$$+\frac{\partial f(\alpha,\sum_j J_{ij}S_j(t))}{\partial\alpha}\delta\alpha+o(\delta\alpha,\delta S). \quad (7)$$

Suppose that there are also deviations in the parameter β and the connecting matrix J; we have

$$\delta S_i(t+1)=\sum_j \frac{\partial f}{\partial S_j}\delta S_i(t)+\frac{\partial f}{\partial\alpha}\delta\alpha+\frac{\partial f}{\partial\beta}\delta\beta$$
$$+\sum_{l,m}\frac{\partial f}{\partial J_{lm}}\delta J_{lm}. \quad (8)$$

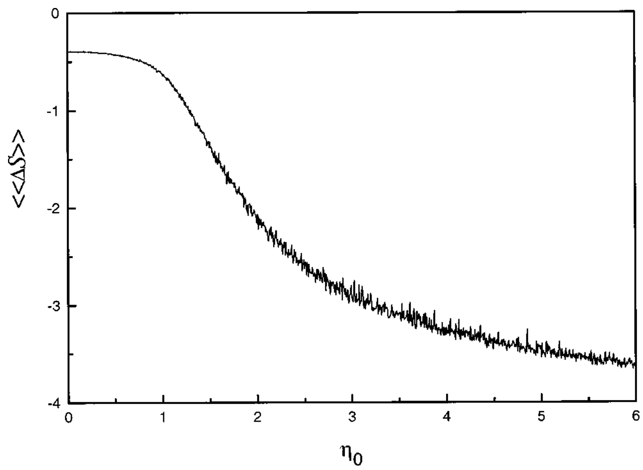

FIG. 3. A plot of noise amplitude η_0 versus the logarithm of the average distance $\langle\langle\Delta S\rangle\rangle$ of the two trajectories when the parameters of the two driven neural networks differ slightly.

One can see that the last three terms will not approach zero even when $\delta S(0)=0$. As a result, synchronization with $\delta S(t>0)\to 0$ is forbidden in all cases, as confirmed by our simulation results. However, if the parameters differ only slightly, the last three terms are very small and so weakly noise-induced synchronization can be observed. Weak synchronization means that the difference of the two trajectories S and S' remains small in average. We define the average distance $\langle\langle\Delta S\rangle\rangle$ of the two trajectories as

$$\langle\langle\Delta S\rangle\rangle=\frac{1}{N}\lim_{T_0\to\infty}\frac{1}{T_1-T_0}\sum_{i=1}^{N}\sum_{t=T_0}^{T_1}|S'_i(t)-S_i(t)|. \quad (9)$$

An example is shown in Fig. 3 with $T_0=500\,000$ and $T_1=501\,000$. Compared with the first network, the slightly different parameters of the second one are $\alpha=6.01$ and $J_{11}=1.01$. From the figure, one can see that the average distance $\langle\langle\Delta S\rangle\rangle$ is smaller than 10^{-3}. It implies that, for sufficiently small deviations in the parameters, the two driven trajectories remain weakly synchronized when the noise amplitude is large enough.

From Fig. 3, one can see that, near the point $\eta_0=1.1$ where finite-precision synchronization is achieved in the case of identical driven systems, the average distance of the two mismatch driven systems is in the order of 0.1. As a result of its positive CLE, the finite-precision synchronization is unstable. Compared with 10^{-18}, the disturbance caused by the mismatch between the two driven systems can be considered as a large noise. As a result, unstable finite-precision synchronization can be easily destroyed by the disturbance. Simulation results also show that, if the noise signal that drives one of the two systems is disturbed by tiny noise, finite-precision synchronization cannot be obtained.

In fact, neural networks are spatiotemporal systems formed by a large number of neurons (e.g., $N\geqslant 100$) that may possess spatiotemporal chaotic behavior. In the rest of this paper, we discuss the possibility of scalar noise-induced synchronization of spatiotemporal chaos. In Refs. [14,30], as a result of the cascade synchronizing dynamics, the spatiotemporal chaos of the coupled logistic map lattice can be driven to a synchronization state by scalar chaos. Different from this, here it is because of the strong synaptic connection that the scalar driving noise can be propagated to the whole network immediately and drive the trajectory of the system to the contracting region with high frequency. As a result, all of the neurons are enforced to the stochastic synchronization state gradually.

The phase-space region in which an attractor resides for chaotic dynamical systems can, in general, be divided into two subregions where a trajectory experiences either pure expansion or pure contraction. In particular, the expanding or contracting region is the region where an infinitesimal vector in the tangent space either expands or contracts under the dynamics. A typical path to synchronization is that the trajectories are driven by a common signal to the contracting region with high frequency, and so the CLE's are negative. In our model, the contracting regions are in the vicinity of the two extreme points and the two asymptotic points of the nonmonotonic neural function $f(x)$, i.e., $df(x)/dx<1$. Now let the first neuron of the neural networks be driven by noise; the weighted input of the ith neuron is

$$h_i(t)=J_{i1}\eta(t)+\sum_{j=1}^{N} J_{ij}S_j(t), \quad i=1,\ldots,N. \tag{10}$$

If all J_{i1}'s ($i=1,\ldots,N$) are nonzero and the noise amplitude η_0 is large enough, due to the random character of connection J, the second term on the right-hand side of Eq. (10) approaches zero when N is very large. It is very small when compared with the first term $J_{i1}\eta(t)$ because $|J_{i1}\eta(t)|$ is large enough with high probability. In other words, the weighted input $h_i(t)$ is mainly determined by the term $J_{i1}\eta(t)$, i.e.,

$$h_i(t)\approx J_{i1}\eta(t). \tag{11}$$

Consider the variational equation for the difference between the two trajectories:

$$\delta S_i(t+1)=f'(h_i)\sum_{j=1}^{N} J_{ij}\delta S_j(t), \tag{12}$$

with

$$f'(h_i)=\exp(-\beta h_i^2)[\alpha \operatorname{sech}^2(\alpha h_i)-2\beta h_i \tanh(\alpha h_i)]. \tag{13}$$

To let $|\delta S_i(t+1)|<|\delta S_i(t)|$, one can see that $\exp(-\beta h_i^2)$ is an important term; i.e., $\beta J_{i1}^2\eta^2(t)$ should be large enough. For a given neural network, a sufficiently large common noise means that the trajectories are driven to the asymptotic contracting region with high frequency. If the overall contraction rate of the driven trajectories is higher than the expansion one in the divergent regime, the distance between trajectories of the systems in the phase space decreases with time, and so the system is transformed to a stable stochastic attractor. In short, if a neuron connected to all other neurons of the neural network is driven by a sufficiently large noise, a stochastic synchronizing state can always be achieved. If all $|J_{i1}|$'s are large enough, noise with small amplitude is capable of driving the networks to synchronization. The asymptotic contracting region is mainly determined by β. A larger β means a wider asymptotic contracting region. The consequence is that noise with a smaller amplitude can also drive the networks to synchronization. To summarize, for a fully connected neural network (i.e., $J_{ij}\neq 0$ for all i,j), a stochastic synchronization state can always be achieved for any neuron driven by noise.

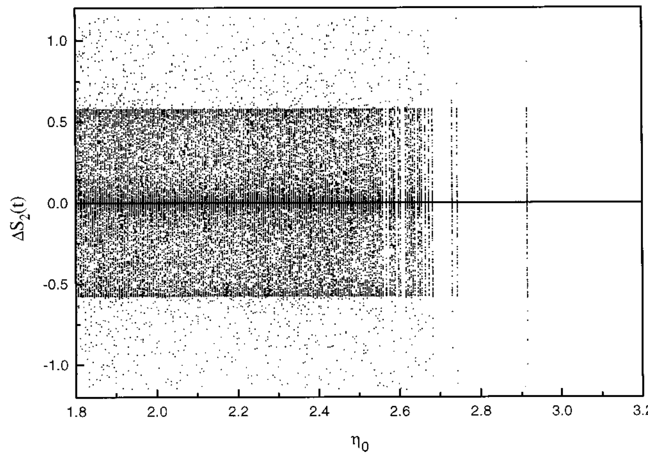

FIG. 4. A plot of noise amplitude η_0 versus the difference between the two second neurons, i.e., $\Delta S_2(t)=S_2'(t)-S_2(t)$. Here for each fixed η_0, 200 $\Delta S_2(t)$'s are drawn with time t from 20 000 to 20 300.

The results of computer simulations confirm these conclusions. In the simulation, the neural network consists of $N=100$ neurons with $\alpha=6.0$ and $\beta=10.0$. The elements of the connecting matrix are randomly selected from the range $0.5<|J_{ij}|<1.0$. Both first neurons of the two identical neural networks are disturbed by a common random noise. For example, Fig. 4 shows a plot of the noise amplitude η_0 versus the difference between the two second neurons, i.e., $\Delta S_2(t)=S_2'(t)-S_2(t)$, with time from 20 000 to 20 300. Simulation results show that when the noise amplitude $\eta_0\geqslant 2.68$, synchronization can be achieved in this example. Figure 5 shows a plot of time versus the logarithm of the average absolute difference $\langle \Delta S(t)\rangle$ between two trajectories with $\eta_0=2.9$. One can see that, at time $t=7400$, all the neurons are in synchronization states and the average absolute difference is smaller than 10^{-18}.

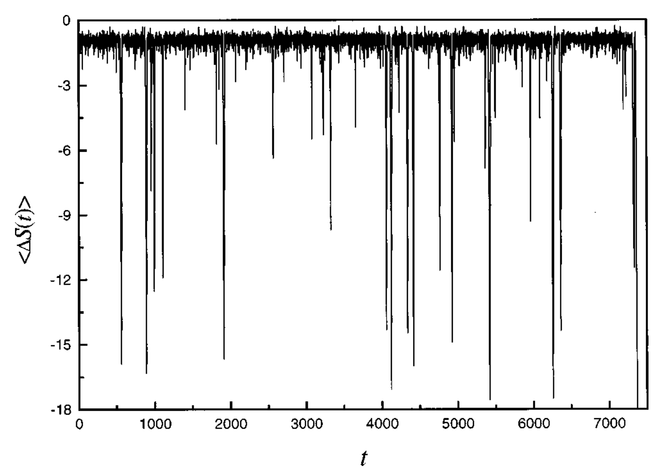

FIG. 5. A plot of time versus the logarithm of the average absolute difference $\langle \Delta S(t)\rangle$ between two trajectories of neural networks with noise amplitude $\eta_0=2.9$.

Now we discuss the situations that some of the J_{i1}'s are zero. The first case is that only $J_{11}=0$. As a result,

$$h_1(t)=\sum_{j=2}^{N} J_{1j}S_j(t),$$

$$h_i(t)\approx J_{i1}\eta(t), \quad i=2,\ldots,N. \tag{14}$$

These equations imply that once the ith neurons S_i (with $i>1$) are driven by a sufficiently large noise to synchronization, the first neuron S_1 is also driven to synchronization.

The situation with some other J_{i1}'s equal to zero is rather complex. The simplest case is that one J_{i1} ($i>1$) equals zero, e.g., $J_{21}=0$. Then we have

$$h_2(t)=J_{22}S_2(t)+\sum_{j=3}^{N} J_{2j}S_j(t), \tag{15}$$

$$h_i(t)=J_{i1}\eta(t)+J_{i2}S_2(t)+\sum_{j\neq 2}^{N} J_{ij}S_j(t), \quad i=1,3,\ldots,N. \tag{16}$$

As the mean of $h_2(t)$ is zero, the term $\exp(-\beta h_2^2)$ has a high probability to approach 1. Thus, unlike the other $\exp(-\beta h_i^2)$ (where $i\neq 2$), the term $\exp(-\beta h_2^2)$ seldom contributes a sufficiently small value to Eq. (13) even when the noise $\eta(t)$ is large enough. As a result, one cannot always obtain $|\delta S_2(t+1)|<|\delta S_2(t)|$ with large enough noise. It implies that, when the two neural networks are driven by a common noise, the initial difference $\Delta S_2(0)=S_2'(0)-S_2(0)\neq 0$ can always have an effect upon $\delta S_2(t)$. Although noise with a large amplitude can drive the neurons S_i ($i\neq 2$) of the two neural networks to approach each other, Eq. (16) shows that the difference $\Delta S_2(t)$ always affects the values of these neurons. One can say that, in the case that some J_{i1}'s are zero, it is difficult to determine whether the two neural networks can be driven to synchronize by the common noise. This depends on the particular connection matrix. Only when all of the CLE's are negative can a stable synchronization state be obtained, as confirmed by the simulation results. A simple example is a network with a connection matrix stated in Eq. (4). Simulation results show that, with the fourth neuron driven by noise (no matter how large the noise amplitude is), the two second neurons of the networks will not approach each other. The CLE associated with the second neuron is always positive.

In summary, in fully connected neural networks, scalar noise can drive low-dimensional hyperchaos or high-dimensional spatiotemporal chaos to a stochastic synchronizing state. When the amplitude of scalar noise is large enough, it can be propagated to all the neurons simultaneously and enforce the networks' trajectory to the contracting regions with high frequency. As a result, trajectories with different initial conditions will be driven to approach each other with high frequency. If the neuron driven by noise is not connected to all other neurons of the network, a synchronization state cannot always be obtained no matter how large the driving noise amplitude is. If the CLE's of the driven system are all negative, they will fall into a stable stochastic synchronization state. When the parameters of the two driven systems differ slightly, weakly noise-induced synchronization can be obtained. Finite-precision synchronization with positive CLE is also observed. It is caused by the on-off synchronizing intermittency of the driven systems. As a result of its unstable character, chaotic finite-precision synchronization cannot be achieved if the two driven systems are mismatched.

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[1] H. Fujisaka and T. Yamada, Prog. Theor. Phys. **69**, 32 (1983).
[2] H. Fujisaka and T. Yamada, Prog. Theor. Phys. **70**, 1240 (1983).
[3] A. S. Pikovsky, Z. Phys. B **55**, 149 (1984).
[4] V. S. Afraimovich, N. N. Verichev, and M. I. Rabinovich, Radiophys. Quantum Electron. **29**, 795 (1986).
[5] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[6] J. P. Eckmann, Rev. Mod. Phys. **53**, 643 (1981).
[7] J. C. Sommerer, W. L. Ditto, C. Grebogi, E. Ott, and M. L. Spano, Phys. Rev. Lett. **66**, 1947 (1991).
[8] H. Herzel and B. Pompe, Phys. Lett. A **122**, 121 (1987).
[9] J. P. Crutchfield, J. D. Farmer, and B. A. Hubermann, Phys. Rep. **92**, 45 (1982).
[10] R. Brown, N. F. Rulkov, and E. R. Tracy, Phys. Rev. E **49**, 3784 (1994).
[11] N. F. Rulkov, M. M. Sushchik, L. S. Tsimring, and H. D. I. Abarbanel, Phys. Rev. E **51**, 980 (1995).
[12] L. Kocarev and U. Parlitz, Phys. Rev. Lett. **76**, 1816 (1996).
[13] H. D. I. Abarbanel, N. F. Rulkov, and M. M. Sushchik, Phys. Rev. E **53**, 4528 (1996).
[14] J. H. Xiao, G. Hu, and Z. Qu, Phys. Rev. Lett. **77**, 4162 (1996).
[15] A. S. Pikovsky, Phys. Lett. A **165**, 33 (1992).
[16] S. Fahy and D. R. Hamann, Phys. Rev. Lett. **69**, 761 (1992).
[17] A. Maritan and J. R. Banavar, Phys. Rev. Lett. **72**, 1451 (1994).
[18] A. S. Pikovsky, Phys. Rev. Lett. **73**, 2931 (1994).
[19] A. Maritan and J. R. Banavar, Phys. Rev. Lett. **73**, 2932 (1994).
[20] H. Herzel and J. Freund, Phys. Rev. E **52**, 3238 (1995).
[21] G. Malescio, Phys. Rev. E **53**, 6551 (1996).
[22] P. M. Gade and C. Basu, Phys. Lett. A **217**, 21 (1996).
[23] L. Longa, E. M. F. Curado, and F. A. Oliveira, Phys. Rev. E **54**, R2201 (1996).
[24] E. Sanchez, M. A. Matias, and V. P. Munuzuri, Phys. Rev. E **56**, 4068 (1997).
[25] M. K. Ali, Phys. Rev. E **55**, 4804 (1997).
[26] L. Kocarev and U. Parlitz, Phys. Rev. Lett. **74**, 5028 (1995).
[27] J. H. Peng, F. J. Ding, M. Ding, and W. Yang, Phys. Rev. Lett. **76**, 904 (1996).

[28] A. Tamasevicius and A. Cenys, Phys. Rev. E **55**, 297 (1997).
[29] Y. C. Lai and C. Grebogi, Phys. Rev. E **50**, 1894 (1994).
[30] J. W. Shuai, K. W. Wong, and L. M. Cheng, Phys. Rev. E **56**, 2272 (1997).
[31] J. W. Shuai, Z. X. Chen, R. T. Liu, and B. X. Wu, Phys. Lett. A **221**, 311 (1996).
[32] J. W. Shuai, Z. X. Chen, R. T. Liu, and B. X. Wu, Phys. Rev. E **56**, 890 (1997).
[33] L. Molgedey, J. Schuchhardt, and H. G. Schuster, Phys. Rev. Lett. **69**, 3717 (1992).
[34] H. L. Yang, Z. Q. Huang, and E. J. Ding, Phys. Rev. E **56**, R2355 (1997).
[35] S. Kim, S. H. Park, and C. S. Ryu, Phys. Rev. Lett. **78**, 1616 (1997).
[36] N. Platt, E. A. Spiegel, and C. Tresser, Phys. Rev. Lett. **70**, 279 (1993).